NC STATE UNIVERSITY

College of Engineering

Department of Mechanical and Aerospace Engineering

MAE-208

Engineering Dynamics

Rube Goldberg Machine – Dynamics Worldwide

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Abstract

This project applied principles of dynamics to the study of a Rube Goldberg machine, a series of contraptions with a complicated design yet simple purpose. This machine was designed and constructed out of household materials with the exception of a small pulley and spring. Data on its motion was collected using multiple cameras to capture both detailed videos of the individual components and an overall picture of the entire process. A complete run-through of the machine can be found here: https://youtu.be/wZYrBoGLx3I).

After constructing and gathering video data from the machine, the videos were analyzed using the physics software Tracker and compared to theoretical estimates made using dynamics principles. Several different dynamics concepts were modeled during this project including conservation of energy, impact, kinetic energy, kinematics, and pully motion. While many of the theoretical predictions were close to their real-world values, many assumptions made in dynamics class were examined to try and explain discrepancies.

1. Introduction

Rube Goldberg studied engineering at UC Berkeley but quit shortly after starting his career. He instead became an illustrator who created a range of cartoons spanning subjects from sports to politics. Rube Goldberg machines became popularized in 1912 with his drawings of complicated machines designed to complete simple tasks. These machines have gone far beyond the paper to be used in cartoons such as Tom and Jerry or in a fascinating way by the band "OK Go," who uses them to create music or to work in time with their music videos.

The purpose for building and analyzing a Rube Goldberg machine in this project was to create an opportunity to experimentally validate many of the concepts we have learned in MAE 208. Many mechanisms used by engineers involve multiple components, which depend on the performance of the prior component. This is especially evident in assembly lines in factories, but can also be seen in cars, and even in circus trapeze acts. It is important to understand how one piece of a machine finishes its task to determine how future pieces can complete theirs. This Rube Goldberg Machine is representative of larger machines, though if it were to be used on a larger scale, it would likely have included more tracks and specifically calibrated equipment to make each run of the machine more uniform.

2. Experimental Methods

After determining what machine, the team wanted to study, an initial brainstorming session was held to come up with individual concepts. It was important that a variety of contraptions were included but also that the components were all able interact appropriately to function as a whole. The team also had to come up with a simple task that could be performed using all of individual parts and the resources available.

After a brief deliberation, it was decided that the Rube Goldberg machine would be created to tell a joke in a very complicated way. The machine is started through the compression of a spring, which launches a foam ball in an elevated position across a straight line to knock a tennis ball down a ramp. This ball lands on a catapult which launches a table tennis ball into a funnel, which then feeds into a cup. The cup then falls down a short distance, using mechanical advantage to lift a sign to and introduce the joke "Why does Pitbull like to open doors?". The cup also triggers a ball to fall down a ramp and knock down a string of dominoes, deck of cards, and two large books. When the last book is knocked over, a rod holding a heavy object (bottle of fabric spray) stationary on a ramp, falls allowing the object to slide down the ramp and causing a door to be opened. Behind the door is a whiteboard with the punchline "He just wants to feel this moment", referencing the popular Pitbull song "Feel this Moment" and the physical moment that must be applied to open a door.

The Rube Goldberg machine was constructed in a garage using household items (See Figure 1). A list of materials can be seen in Table 1. For a more complete image of the setup, refer to the video linked in the abstract. Data on machine performance was collected using video recordings and analyzed using the physics analysis software Tracker 6.0.3. Two types of recordings were made. One was a wide angle shot of the entire machine as each component operated sequentially so as to demonstrate the general operation of the entire machine. The other type of recording was a close-up video detailing the operation of each of the components. These recordings were used to compare dynamics theories to experimental results and are what provided the data for the Data and Analysis section.

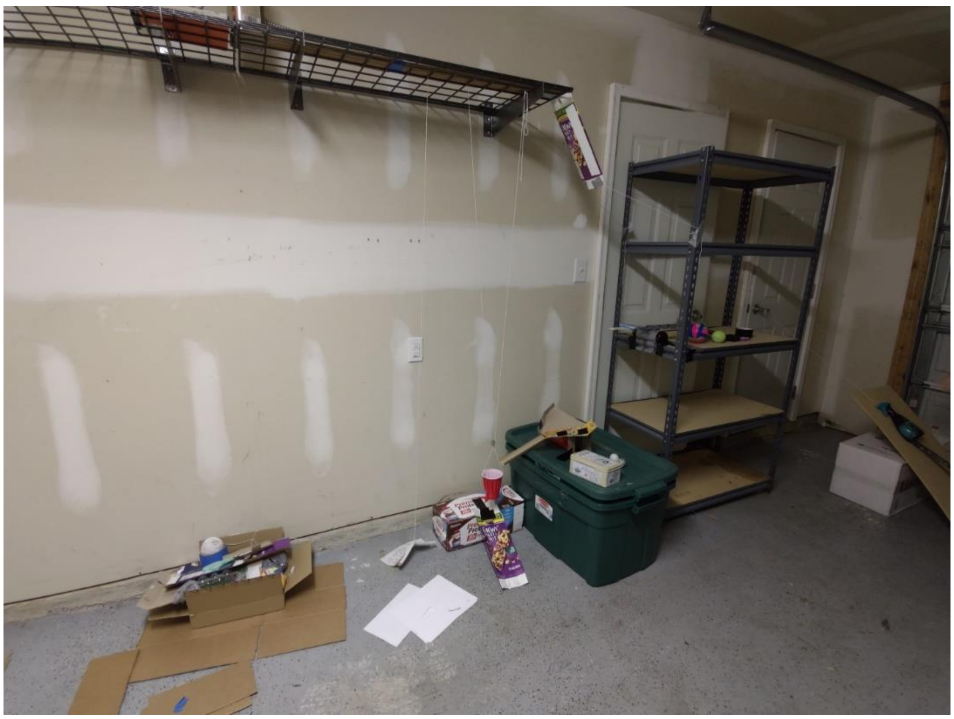

Figure 1: Machine Overview

Table 1: Materials Used

Materials Used		
Small Nickel Pulley	Spring	All-purpose twine
Table Tennis Ball	2x Foam Toy Ball	Red SOLO™ Cup
Cardboard Boxes	Duct Tape	Dominoes
Deck of Cards	2x Textbooks	Bottle of Fabric Spray
Large Kitchen Spoon	Measuring Stick	White Fabric
Whiteboard	Markers	1 Gal. Olive Oil Can

Table 2: Variables Used

Variables Used in Calculations	
Variable Symbol	**Meaning**
v	velocity
m	mass
s	distance
t	time
a	acceleration
k	spring constant
g	gravity
F	Force
θ	Angle

3. Experimental Data and Analysis

For each of the sections below, theoretical calculations were completed using known values and compared to actual numbers found using the Tracker app. Tracker allowed us to upload shots of each of the components and track their motion to find distances, velocities, and accelerations. One important part of this project was the placement of 1 ft tape marks that could be seen in each close up shot. This allowed for the correct calibration of distances after the videos were uploaded into the software to be analyzed.

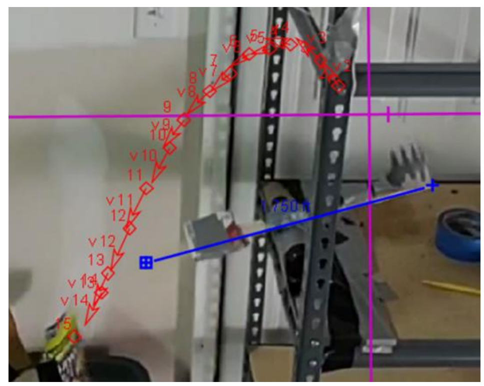

Figure 2: Projectile Tracking

3.1. Spring

Assumptions: Spring and Surface are frictionless; ball does not deflect.

Prior to beginning analysis on the spring start, the spring constant k was determined through Hooke's law F=kx. A water bottle with a known mass [g] was attached to a vertical unstretched spring until stretched to a new equilibrium. Using the acceleration due to gravity 32.2 and the mass converted to slugs using the conversion factor [1g = 6.85*10^-5slugs] in the calculation of the weight as the force and the measured change in length of the spring, k can be determined.

Goal: Determine the velocity of the ball right after it leaves the spring

Force of Waterbottle: $F_{Wb} = m_{Wb} * a_g = (1.09 * 10^3 g)\left(\frac{6.85 * 10^{-5}\ slugs}{1\ g}\right)\left(32.2\frac{ft}{s^2}\right) = \mathbf{2.4}\ \boldsymbol{lbs}$

Spring Constant: $F = k * \Delta x \rightarrow k = \frac{F_{Wb}}{\Delta x} = \frac{2.4\ lbs}{0.05\ ft} = \mathbf{48}\frac{\boldsymbol{lbs}}{\boldsymbol{ft}}$

Conservation of Energy: $\frac{1}{2}k(s_2^2 - s_1^2) = \frac{1}{2}m_A v_A^2$

$\frac{1}{2}\left(48\frac{lbs}{ft}\right)((0.33ft - 0.236ft)^2 - (0)^2) = \frac{1}{2}(0.00356\ slugs)(v_A^2)$

$v_A(theoretical) = \mathbf{5.68}\frac{\boldsymbol{ft}}{\boldsymbol{s}}$

$v_A(actual) = \mathbf{2.76}\frac{\boldsymbol{ft}}{\boldsymbol{s}}$

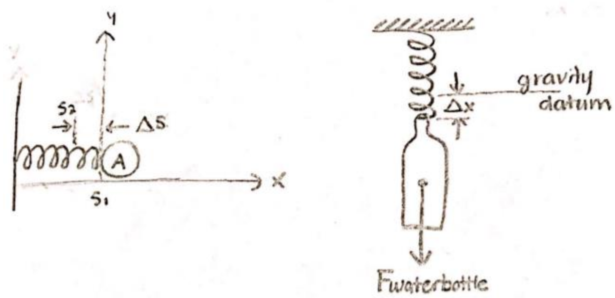

Figure 3: Determining K and Spring Setup

The calculated velocity was 5.68 ft/s, which was larger than the experimental value of 2.77 ft/s. This difference could be explained by several things including the friction of the spring along with the limitation of using a hand to release the ball. The force of friction in the spring or on the surface of the ball along with the force from the hand if it was not removed following the release of the ball would result in work opposing the direction of motion, which would result in less kinetic energy when the ball is released from the spring and a smaller velocity. Thus, after all of the assumptions that had to be made, this equation was not a very good model for predicting the speed of the ball.

3.2. Conservation of Momentum

Assumptions: frictionless, there are no external forces acting on the balls, and both balls deflect, adding to the coefficient of restitution.

Goal: Determine the coefficient of restitution using the experimentally determined initial and final velocities of each ball.

Table 3: Experimentally Determined Initial and Final Ball Velocities from Tracker

Ball	Velocity Before Impact	Velocity After Impact
A	1.55 ft/s	1.55 ft/s
B	0 ft/s	-0.51 ft/s

$$e = \frac{v_{B2} - v_{A2}}{v_{A1} - v_{B1}} = \frac{0.6 - (-0.51)}{1.55 - 0} = 0.72$$

The calculated coefficient of restitution was e = 0.72, which makes sense as the balls did elastically bounce off one another, but no realistic system is perfectly elastic.

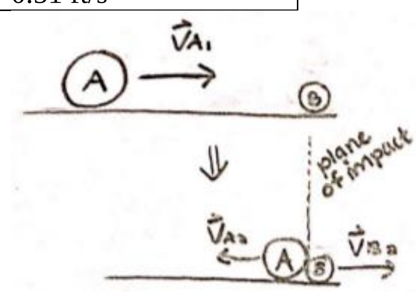

Figure 4: Impact of Balls A and B

3.3. Ramp

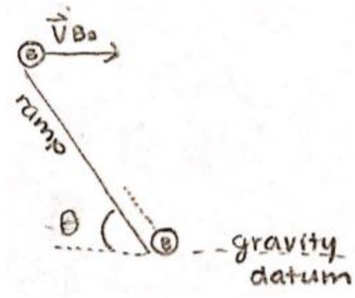

Figure 5: Tennis Ball on Ramp

The final velocity of a ball rolling down a ramp can be estimated in multiple ways. Here, kinetics and conservation of energy methods were used and compared to experimentally obtained values.

3.3.1. Kinetics Method

Similar to the example on page VIII.27 of the MAE 208 Course Pack, a free body diagram was used to create equations relating the forces on the ball and its acceleration.

$$\sum F_x = ma_{Gx} = mgsin(\theta) - F_f \qquad \sum M_G = I_G\alpha = -F_f r \qquad a_{Gx} = -\alpha r \qquad I_G = \frac{2}{3}mr^2$$

$$mgsin(\theta) - \frac{I_G a_{Gx}}{r^2} = ma_{Gx} \qquad gsin\theta - \frac{2}{3}a_{Gx} = a_{Gx} \qquad a_{Gx} = \frac{3}{5}gsin\theta$$

Using $v_f = v_i + at$, $\quad v_f = 0.7487\ \frac{ft}{sec} + \frac{3}{5}\left(32.2\ \frac{ft}{sec^2}\right)\sin(55°)(0.46\ sec) = \mathbf{8.03\ \frac{ft}{sec}}$

3.3.2. Conservation of Energy Method

The second method used a conservation of energy approach, where the initial kinetic energy and potential energy of the ball was equated to the kinetic energy of the ball at the end of the ramp. It should be noted that this method is expected to produce error, as it does not account for losses due to friction. Using the conservation of energy method, the final velocity of the ball at the end of the ramp was calculated to be

$$T_1 + V_1 = T_2 + V_2 \qquad \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 + mgh = \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 \qquad v = r\omega \qquad I = \frac{2}{3}mr^2$$

$$\frac{5}{6}r^2\omega_1^2 + gh = \frac{5}{6}r^2\omega_2^2 \qquad \omega_1^2 + \frac{6gh}{5r^2} = \omega_2^2 \qquad v_2 = r\sqrt{\left(\frac{v_1}{r}\right)^2 + \frac{6gh}{5r^2}}$$

$$v_2 = (0.108\ ft)\sqrt{\left(\frac{0.7487\ \frac{ft}{sec}}{0.108\ ft}\right)^2 + \frac{6\left(32.2\ \frac{ft}{sec^2}\right)(1.274\ ft)}{5(0.108\ ft)^2}} \qquad \mathbf{v_2 = 7.06\ \frac{ft}{sec}}$$

The experimentally obtained value for the final velocity was $\mathbf{v_2 = 6.23\ \frac{ft}{sec}}$. The discrepancies between the theoretical and experimental results are most likely caused by errors in the measured variables inputted into the equations, losses due to rolling friction, and an assumed value for mass moment of inertia.

3.4. Catapult

Assumptions: No air resistance, acceleration is constant (gravity is the only force acting on the ball).

Goal: Determine the max height and x distance traveled by the ball using the initial velocities in x and y.

$$v_y = v_{iy} * -gt$$

$$t = \frac{v_{iy}}{g} = \frac{4.377}{32.2} = .1359s$$

$$s_y = s_o + v_{iy} - \frac{1}{2}gt^2 = 0 + 4.377(.1359) - \frac{1}{2}(32.2)(.1359)^2 = \mathbf{0.297\ ft}$$

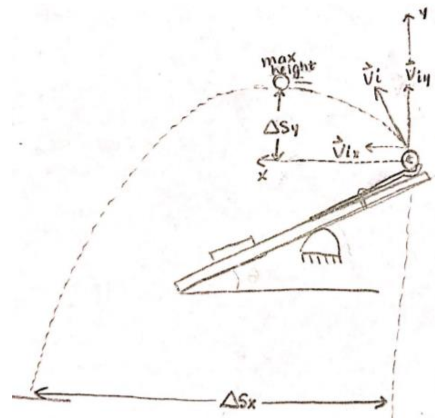

Figure 6: Catapult Setup and Variables Used

$$\Delta s_x = v_{ix} * t = 3.55(0.5) = \mathbf{1.77 ft}$$

The calculated max height was 0.297 ft while the experimental height was 0.284 ft. This likely accounts for air resistance or the oatmeal and rocks not being evenly distributed throughout the ball causing the flight trajectory to be slightly off. The calculated x distance was also larger at 1.77 ft compared to the actual 1.742 ft. This can be explained by the same reasons as above.

3.5. Pulley

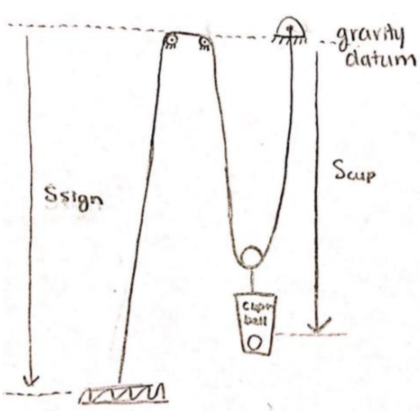

Figure 7: Pulley Setup

The cup was limited to fall through a short distance. To lift the sign high enough to be read, the pulley utilizes mechanical advantage to lift the sign nearly double the distance that the cup fell through.

Assumptions: Pulley is massless and frictionless; tension is the same throughout all cables, and cables do not stretch.

Goal: Lift sign to introduce joke. Determine how close the relationships introduced in class hold up in the real world when the above assumptions cannot be followed by finding the relationship between the distance traveled of the cup and the sign and the velocities of each. For both the distance traveled and the velocities, the values shown are only in the y direction.

Table 4: Cup and Sign Relationships used in Pulley Problem

Variable	For the Sign	For the Cup	Theoretical Relationship	Actual Relationship
Distance	1.054 ft	-0.522 ft	$s_{sign} = -2s_{cup}$	$s_{sign} = -2.02s_{cup}$
Velocity	.8 ft/s	-.31 ft/s	$v_{sign} = -2v_{cup}$	$v_{sign} = -2.5v_{cup}$

By mechanical advantage, the velocity and distance traveled by the sign should be approximately double that of the cup. In class, assumptions were made such that the pulley was massless and frictionless, tension in the cables was constant, and the cables do not stretch.

When the relationship between the total distance traveled for the cup and the sign was compared, it was found the theoretical value was almost exactly replicated. This made sense because the string was relatively stiff, and the masses of the objects attached were small so the relationship of the distances could not have changed by a large factor.

When the relationships between the velocities were compared, it was found that the y velocity of the cup was slightly lower than what it should have been if the assumptions held up. The most reasonable explanation for this result is that the ball had an initial x velocity along with its y velocity. This means that, some of the momentum rocked the cup in the x direction rather than being conserved solely in the y direction. The velocity for the sign was almost completely in the y direction, allowing it to move more quickly in that direction than the cup.

Typical class problems involve velocities in one direction. To take a closer look at the above theory, the magnitude of the cup's entire velocity was examined using Tracker. The approximate magnitude of the cup's initial velocity was around -0.42. making the relationship $v_{sign} = -1.9v_{cup}$, which is more in line with what would be expected from class assumptions.

3.6 Theoretical and Actual Value Comparison

After the calculations and values were measured, a percent error formula was used to compare the theoretical and experimental numbers for each examined variable. While the discrepancies for each calculation were discussed in the individual sections, this was a way to show the true magnitude of these errors while summarizing all of the values against each other. The percent error formula and a table showing these results are included below.

$$\boldsymbol{Percent\ Error} = \left|\frac{\boldsymbol{Actual - Theoretical}}{\boldsymbol{Theoretical}}\right| * \mathbf{100}$$

Table 5: Percent Error Comparison

Component	Variable	Theoretical Value	Actual Value	Percent Error
Spring	Va	5.68	2.76	51.4%
Ramp	Vf1	8.03	6.23	22.4%
	Vf2	7.06	6.23	11.8%
Catapult	Δsy	0.297	0.284	4.4%
	Δsx	1.77	1.742	1.6%
Pulley	S ratio	2	2.02	1.0%
	V ratio 1	2	2.5	25.0%
	V ratio 2	2	1.9	5.0%

4. Conclusion

The project was successful in completing the desired task of opening a door and telling the joke. It was able to cover a broad scale of course topics including conservation of energy, conservation of momentum, dependent pulley motion, and kinematics. It was difficult with the limited cameras available to capture good enough videos to analyze each step in the machine using the same trial. To get good angles and videos to use in Tracker, each step in the machine was recorded separately and the experimental initial velocities were used. Some assumptions and approximations were also used that may not be very true to real life. Things are not frictionless, and they do not have straight line vectors. The pulley can be very complex to analyze when the projectile entering it does not have velocity in only one direction. Analysis did utilize generalized models, which achieved close approximations to the experimental results. For anything requiring more consistency, a more calibrated machine and in-depth models and analysis would be required.

The construction of the machine itself presented several challenges. It involved three trips to obtain different balls, duct tape, pulleys, springs, and other non – household items. There was a decent amount of planning that went into the materials list, but during construction, it was found that more items would be useful. The most complications came from the catapult and pulley especially when working in conjunction. The catapult was first constructed using tape, but continuously fell and lacked consistency. This was improved using rubber bands and markings on how to set up the catapult after a trial. Another issue came from the tennis ball also being launched into the funnel. This may have been avoided if the catapult was not so close to the funnel but would have required a heavier ball to launch the oatmeal filled ping-pong ball a greater distance. The pulley was initially created with household items, but due to friction between components, it would constantly get twisted. This was improved using a small bait and tackle pulley, which decreased friction between the string and the pulley, though it was heavier, which required that the ball be filled with oatmeal.

The filming required several takes due to anything from the dominoes falling too soon or due to the complications with construction mentioned above. However, the overall project was fun and interesting. The brainstorming process presented several opportunities to tie different dynamics concepts into the construction of the machine as well as in the development of the objective, which allowed the project to tie in a Dynamics related joke.

5. References

Norman Rockwell Museum. (n.d.). *Rube Goldberg*. Illustration History. Retrieved November 23, 2021, from https://www.illustrationhistory.org/artists/rube-goldberg.